

## Development of Functional Relationships for Air-Data Estimation using Numerical Simulations

S. A. Razzaqi<sup>1</sup>, C. Bode<sup>2</sup>, T. Eggers<sup>2</sup> and M. K. Smart<sup>1</sup>

<sup>1</sup>Centre for Hypersonics, School of Mechanical and Mining Engineering  
The University of Queensland, Brisbane, Queensland 4072, Australia

<sup>2</sup>Institute of Aerodynamics and Flow Technology, Spacecraft  
DLR Braunschweig, 38108 Braunschweig, Germany

### Abstract

Flush Air-Data Sensing (FADS) systems use an array of surface pressure measurements to infer the speed, position, and orientation of a vehicle in flight. The non-intrusive nature of such systems make them especially useful for hypersonic vehicles. Determining the functional relationship between measured surface pressure and the airdata of interest (e.g. Mach number, angle-of-attack, etc.) is the first step in implementing a FADS system. In this work, these functional relationships are developed by fitting surfaces/curves to a set of numerical simulations. In the absence of ground or flight test data, the resulting FADS algorithm is tested with a second set of numerical simulations. The largest error in predicted Mach number is  $\pm 0.2$ , in angle-of-attack is  $\pm 0.05^\circ$ , in angle-of-sideslip is  $\pm 0.15^\circ$ , and in freestream pressures is  $\pm 20$  Pa.

### Introduction

Scramjet powered vehicles are expected to be an integral part of future access-to-space systems [1, 2]. Flight experiments are an important component in the continuing development of scramjet technologies. These flight experiments can be controlled flights, such as Hyper-X [3], or uncontrolled flights, such as HyShot II [4]. In the former case, accurate sensing of the vehicle state is essential to the success of the flight control algorithms. And in both cases, characterising the inflow to the scramjet is essential to successfully analysing engine performance.

For a hypersonic vehicle, measurement of vehicle speed, position, and orientation must be accomplished with non-intrusive sensors [5]. These often include Inertial Measurement Units (IMU), Global Positioning Systems (GPS), and Flush Airdata Sensing (FADS) systems. FADS systems can potentially provide real-time vehicle orientation information that is more accurate than that from a typical IMU [6] and also provide post-flight Mach number and altitude estimates to compliment an IMU and/or GPS.

FADS systems use an array of surface pressure measurements to infer important airdata parameters. In particular, in the current work we consider a system capable of predicting Mach number ( $M$ ), freestream pressure ( $P_\infty$ ), angle-of-attack ( $\alpha$ ), and angle-of-sideslip ( $\beta$ ). From these four parameters one can calculate all other airdata parameters of interest. In this paper, a set of numerical simulations is used to develop the necessary relationships between the four relevant airdata parameters and the surface pressure measured at discrete locations on the vehicle.

### FADS Algorithm

The basic FADS algorithm is summarised quite succinctly by equation 1

$$P_i = F_i(M, P_\infty, \alpha, \beta) \quad (1)$$

where  $P_i$  is the  $i^{\text{th}}$  surface pressure measurement and  $F_i$  is a function relating the  $i^{\text{th}}$  surface pressure measurement to  $M$ ,  $P_\infty$ ,  $\alpha$ , and  $\beta$ . In general, one could add an error term  $\epsilon$  to the right-hand side of equation 1 to account for systematic error.

An expression for  $F_i$  can come from an analytical model (e.g. modified Newtonian flow, oblique shock theory etc.), as is the case in most FADS work [7, 8, 9]. The analytical model is then “refined” using some combination of numerical simulations, ground test data, and flight test data.

Alternatively, in this work,  $F_i$  is obtained directly by fitting polynomial surfaces/curves (in  $M$ ,  $P_\infty$ ,  $\alpha$ , and  $\beta$ ) to a set of CFD data. This is a more flexible approach and is well suited to the techniques used to solve equation 1. However, one must be confident that the numerical simulations accurately model the physics of the flow of interest. The approach is also limited to small ranges of  $M$ ,  $P_\infty$ ,  $\alpha$ , and  $\beta$  i.e. a range over which a polynomial functional relationship will hold.

However it is obtained,  $F_i$  is generally highly non-linear. It is also common practice to use more than four discrete pressure measurements. So given  $n$  pressure measurements, equation 1 becomes an overdetermined system of non-linear equations to be solved for  $M$ ,  $P_\infty$ ,  $\alpha$ , and  $\beta$ . The system is often solved by linearizing the equations about a point and solving the system of perturbation equations with a least squares method [8, 9].

The use of these techniques can be problematic, particularly for real-time calculation of airdata parameters; excessive noise or signal drop out can lead to unconverged solutions. As a result, fault management schemes have been developed [5]. Alternatively, one can formulate the problem in a different manner, as the authors in [7] have done with their “triples algorithm”, which avoids non-linear regression.

In the current work, we consider the use of FADS for post-flight analysis of data, and therefore use of non-linear regression techniques is deemed acceptable. A built in MATLAB function is used to solve equation 1 directly (the system is linearised, but this step is taken care of by the MATLAB function rather than the FADS algorithm).

### Vehicle Geometry and Pressure Port Configuration

The vehicle geometry investigated in this work is pictured in figure 1. This geometry is considered to be a sharp-nosed vehicle (though it has a leading edge radius of 1 mm) and is similar to that studied in [6]. This configuration would be ideal for the integration of planar scramjets or shape-transitioning scramjets such as the REST engine [2]. The names and locations of pressure measurement ports are indicated on figure 1 and summarised in table 1.

### Numerical Simulations

The FADS system described in this paper is developed exclu-

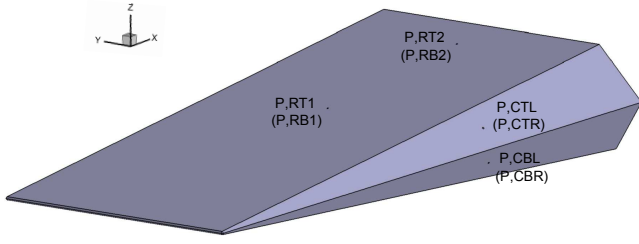


Figure 1: Vehicle geometry and pressure port locations. R = ramp surface, C = chine surface, T = top, B = bottom, R = right, L = left. Pressure ports are distributed symmetrically. Names in brackets give the name of the pressure port on the “opposite” side of the body (i.e. across the relevant symmetry plane).

Port Name	$x$ (m)	$y$ (m)	$z$ (m)
P,RT1	0.150	0.000	0.016
P,RT2	0.277	0.000	0.029
P,RB1	0.150	0.000	-0.016
P,RB2	0.277	0.000	-0.029
P,CTR	0.175	0.109	0.009
P,CTL	0.175	-0.109	0.009
P,CBR	0.175	0.109	-0.009
P,CBL	0.175	-0.109	-0.009

Table 1: Pressure port locations.

sively using numerical simulations of the vehicle. As noted, this requires that the simulations accurately capture the physics of the real flow. For this preliminary work the focus is on demonstrating that a well performing FADS system can be developed using only CFD and the straight-forward surface/curve fitting approach. Also, since there is no flight or ground test data to compare with, CFD simulations are used to both design and test the FADS system. In other words, at this stage of the work, the “real” flow is not that of a notional flight experiment, but rather is defined by a set of numerical simulations used to test the FADS system.

All numerical simulations were carried out by Christoph Bode at the Institute of Aerodynamics and Flow Technology of the DLR in Braunschweig using the DLR TAU code. Meshes were generated with the Centaur Grid Generator. All simulations are inviscid and use an unstructured surface and volume mesh with 1193248 nodes. While inviscid simulations would not accurately model results from a flight experiment, they are acceptable for the goals of this work. Further details on the flow solver and meshing tools are available in [10].

As noted, two sets of simulations were created; one to design the FADS system, and a second to test it. The airdata parameters used as inputs for both the design and test data sets are summarised in table 2. The parameter ranges are based on typical bounds for a sounding rocket flight experiment similar to HyShot II.  $\alpha$  and  $\beta$  are positive in the positive  $z$ -direction and  $y$ -direction, respectively.

### Surface/Curve Fits

The FADS system is developed by performing surface/curve fits to the design data set. This involves specifying the form of  $F_i$ ; in this work  $F_i$  is defined by equation 2 as

$$F_i = (c_{1,i} + c_{2,i}\alpha + c_{3,i}\beta + c_{4,i}\alpha^2 + c_{5,i}\alpha\beta + c_{6,i}\beta^2 + c_{7,i}\alpha^3 + c_{8,i}\alpha^2\beta + c_{9,i}\alpha\beta^2 + c_{10,i}\beta^3)P_\infty \quad (2)$$

	$M$	$\alpha$ ( $^\circ$ )	$\beta$ ( $^\circ$ )	$P_\infty$ (Pa)
Design Data Set	7.0, 7.5, 8.0, 8.5	0, 1, 2, 4	0, 1, 2, 4	1181
Test Data Set	7.25, 7.86, 8.15, 8.39	-2.7, -0.5, 2.1	-3.0, 0.5, 1.5, 3.1	559

Table 2: CFD data sets. Design data set: 64 simulations. Test data set: 48 simulations

with  $i = 1 \dots 8$  (the number of pressure ports) and where  $c_{j,i}$  (with  $j = 1 \dots 10$ ) are functions of Mach number. The assumed functional relationship for  $c_{j,i}$  is given by equation 3 as

$$c_{j,i} = A_{j,i}M^2 + B_{j,i}M + C_{j,i} \quad (3)$$

So, for each design Mach number ( $M = 7.0, 7.5, 8.0, 8.5$ ) discrete values for  $c_{j,i}$  are obtained by fitting  $F_i$  to the surface pressure data  $P_i$  from the design data set. A representative surface fit is shown in figure 2. Then, the constants  $A_{j,i}$ ,  $B_{j,i}$ , and  $C_{j,i}$  are obtained by fitting equation 3 to the values of  $c_{j,i}$  at each design Mach number as obtained in the previous step.

$F_i$  is a polynomial function in  $M$ ,  $\alpha$ ,  $\beta$ , and  $P_\infty$ ; however, it is easier to work with in the manner specified (i.e. by first performing a surface fit in  $\alpha$  and  $\beta$ , then a fit to the constants in  $M$ ), rather than dealing with  $F_i$  as a whole at once.

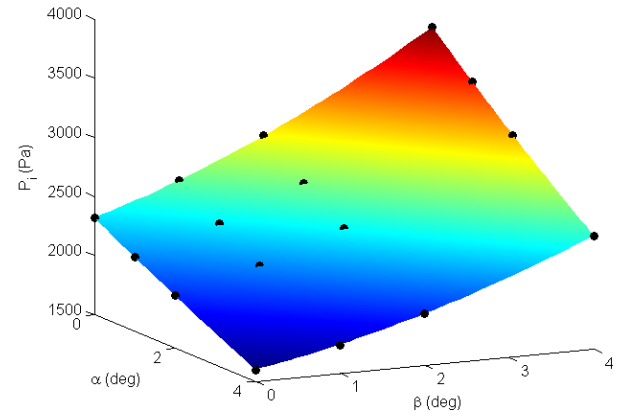


Figure 2: Representative surface fit for  $M = 8.5$  and port P,CTL.

The quality of the surface fit for  $F_i$  ultimately determines the accuracy of the FADS algorithm. Over the parameter range of interest the maximum residual is 0.002 (corresponding to  $\sim 3$  Pa), and generally it is an order of magnitude less. This should be sufficient to obtain good airdata estimates.

### Airdata Parameter Prediction

The non-linear system specified by equations 1, 2, and 3 is solved using the Levenberg-Marquardt (LM) algorithm that is built into MATLAB. For each data point in the test data set, the LM algorithm is run one hundred times, each time from a randomly generated start point lying within the bounds  $[-10, 10]$  for each parameter. This is not the most efficient optimisation technique, but it has proved robust thus far, resulting in a converged solution for all test data points.

Results for the prediction of  $M$  are shown in figure 3 versus  $\alpha$  and for each value of  $\beta$ . The actual value of  $M$  is indicated by the dashed line. The prediction of  $M$  is very good ( $< \pm 0.05$  or  $\pm \sim 0.7\%$ ) for  $\alpha = -0.5, 2.1$  and less so ( $< \pm 0.2$  or  $\pm \sim 3.0\%$ ) for

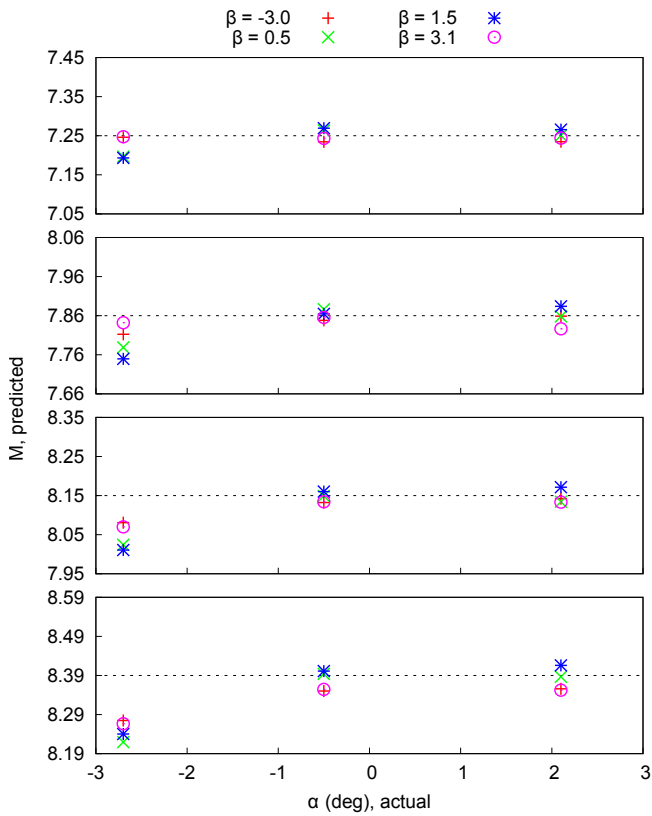


Figure 3: Predicted Mach number.

$\alpha = -2.7$ . For low  $\alpha$ , pressure on the ramp surfaces is essentially independent of  $\beta$ , but for higher (absolute) values of  $\alpha$  this is no longer true. This is why the error in predicted  $M$  is larger for the highest  $|\alpha|$  value and why there is a greater dependence on  $\beta$  at the highest  $|\alpha|$ .

The prediction of  $\alpha$  is shown in figure 4 versus  $M$  and for each value of  $\beta$ . The actual value of  $\alpha$  is given by the dashed line. Again, the prediction is worst for the highest  $|\alpha|$ . The error in predicted  $\alpha$  also increases with increasing  $M$ , likely due as well to the increased coupling of  $\alpha$  and  $\beta$ . However, for all the test data points, the error in predicted  $\alpha$  is  $< \pm 0.05^\circ$  or  $\pm \sim 1.5\%$ .

Predictions of  $\beta$  are shown in figure 5 versus  $M$  and for each value of  $\alpha$ , with the actual value indicated by the dashed line. As before, error in  $\beta$  is larger for higher  $|\alpha|$ . However, interestingly,  $\beta$  is predicted most poorly for the lowest actual value of  $\beta$ . This could result from the fact that the surface pressure is less sensitive to changes in orientation when  $\beta$  is small. The predicted value of  $\beta$  is within  $\pm 0.15^\circ$  for all test data points (equal to  $\pm \sim 2.0\%$  for  $\beta > 0.5$  and  $\pm \sim 22.0\%$  when  $\beta$  is small).

Finally, predictions of  $P_\infty$  are shown in figure 6 versus  $M$  and for each  $\alpha$  and  $\beta$ . All the predictions are greater than the actual value (indicated by the dashed line) of  $P_\infty = 559$  Pa. The design data set simulations were all conducted at the same  $P_\infty$ , which likely skews the functional relationship between surface pressure and freestream pressure. Consistent with earlier results, the error in  $P_\infty$  is largest for higher  $|\alpha|$ . Still, all predictions are within  $\pm 20$  Pa (or  $\pm \sim 4.5\%$ ) of the actual value.

The discussion thus far has essentially quantified the uncertainty in the prediction of airdata parameters due to the quality of the surface fit. Also important is evaluating the sensitivity of the predictions to uncertainty in the measured surface pressure. This can be done by perturbing each input surface pressure in-

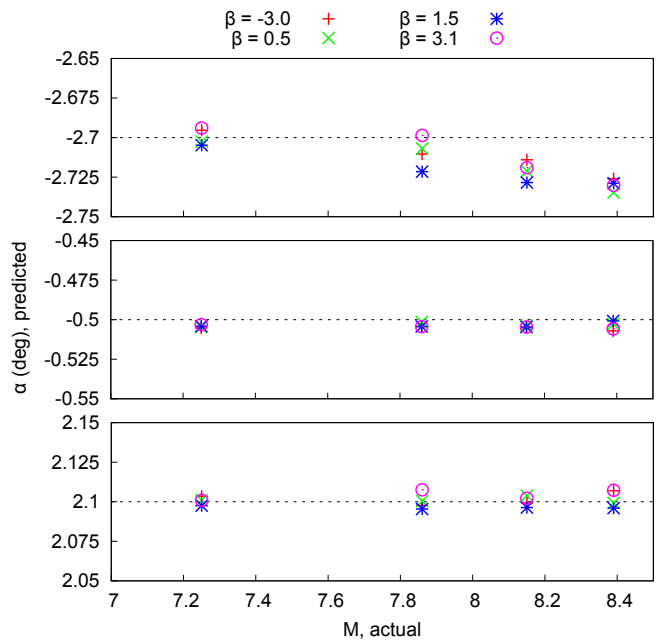


Figure 4: Predicted angle-of-attack.

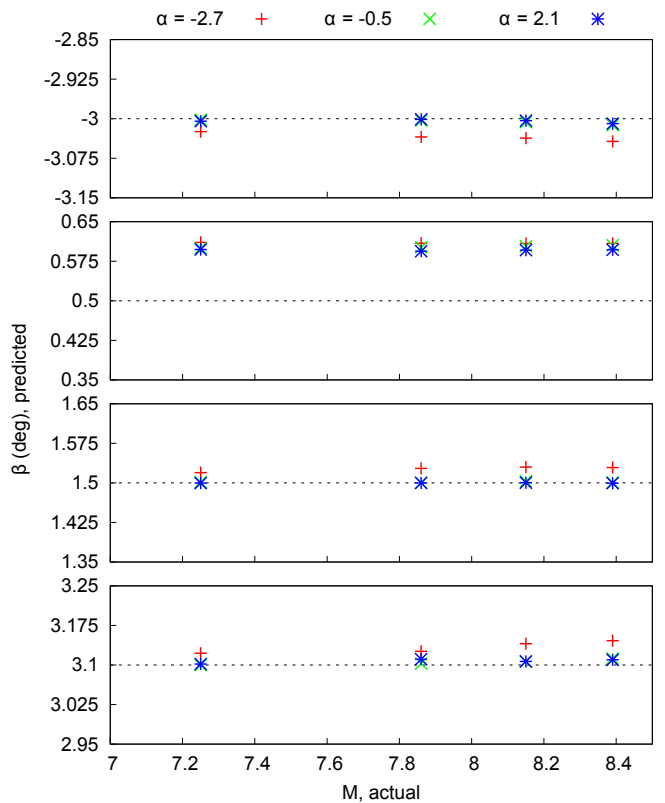


Figure 5: Predicted angle-of-sideslip.

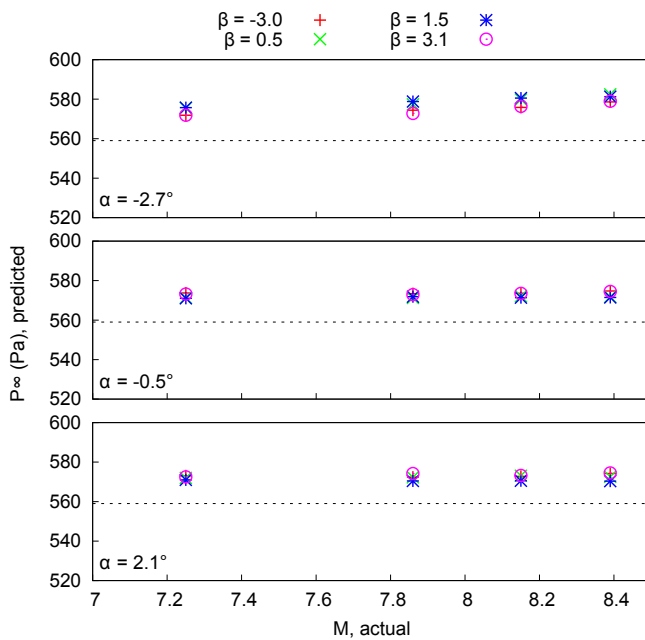


Figure 6: Predicted freestream pressure.

dividually and assessing the relative change in the predicted air-data parameters. For a  $\pm 5\%$  uncertainty in  $P_i$ , the uncertainty in  $M$ ,  $\alpha$ , and  $P_\infty$  is  $< \pm 0.07\%$  and in  $\beta$  is  $< \pm 0.3\%$ .

The robustness of the FADS algorithm (particularly the LM algorithm) to large errors in input values is yet to be determined.

## Conclusions

The success of a FADS algorithm depends largely on the quality of the model used to relate surface pressure to the airdata parameters of interest. In this work, it has been established that fitting surfaces/curves to a set of numerical simulations is a simple technique that can be used to generate this model and that the resulting FADS algorithm, when tested with numerical simulations, performs well over the chosen range of airdata parameters. To develop a FADS system for a flight vehicle the numerical simulations used for design must accurately model the flight conditions. Then, the surface/curve fitting techniques described in this paper can be successfully applied to the design of a FADS system for a hypersonic vehicle.

## References

- [1] L.H. Townend, "Domain of the Scramjet," *Journal of Propulsion and Power*, vol. 17, no. 6, pp. 1205-1213, 2001.
- [2] M.K. Smart and M.R. Tetlow, "Orbital Delivery of Small Payloads using Hypersonic Airbreathing Propulsion," *Journal of Spacecraft and Rockets*, vol. 46, no. 1, pp. 117-125, 2009.
- [3] E. Baumann, J.W. Pahle, M.C. Davis and J.T. White, "The X-43A Flush Airdata Sensing System Flight Test Results," AIAA Atmospheric Flight Mechanics Conference and Exhibit, Honolulu, U.S.A., 2008.
- [4] M.K. Smart, "Flight Data Analysis of the HyShot 2 Scramjet Flight Experiment," *AIAA Journal*, vol. 44, no. 10, pp. 2366-2375, 2006.
- [5] S.A. Whitmore and T.R. Moes, "Failure Detection and Fault Management Techniques for a Pneumatic High-

Angle-of-Attack Flush Airdata Sensing (HI-FADS) System," NASA Technical Memorandum 4335, 1992.

- [6] M.C. Davis, J.W. Pahle, J.T. White, L.A. Marshall, M.J. Mashburn and R. Franks, "Development of a Flush Airdata Sensing System on a Sharp-Nosed Vehicle for Flight at Mach 3 to 8," NASA Technical Memorandum 2000-209017, 2000.
- [7] S.A. Whitmore, B.R. Cobleigh and E.A. Haering, "Design and Calibration of the X-33 Flush Airdata Sensing (FADS) System," NASA Technical Memorandum 1998-206540, 1998.
- [8] S.A. Whitmore and T.R. Moes, "Measurement Uncertainty and Feasibility Study of a Flush Airdata System for a Hypersonic Flight Experiment," NASA Technical Memorandum 4627, 1994.
- [9] I.A. Johnston, P.A. Jacobs and T. Shimoda, "Flush Air Data System Calibration Using Numerical Simulations", *Journal of Spacecraft and Rockets*, vol. 35, no. 6, pp. 812-820, 1998.
- [10] C. Bode, "Numerical Generation of a Flush Air Data System for the Hypersonic Flight Experiment HIFiRE 7," Diploma-Thesis, DLR Braunschweig and The University of Queensland, 2010.